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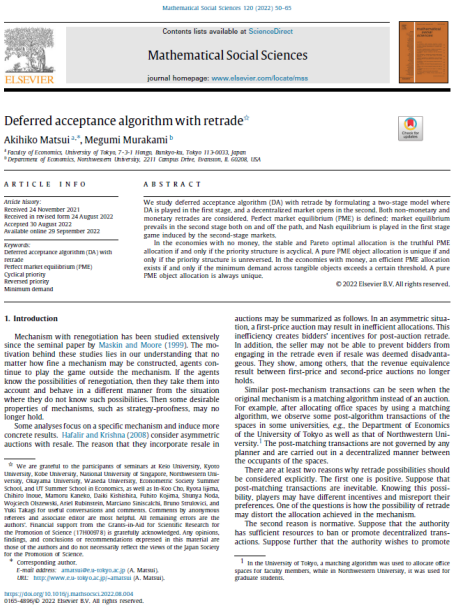
学術論文と一口に言っても分野によって投稿までの時間、執筆人数は大きく異なると聞きます。経済学の場合は、単著ないし２から３人の研究が多く、構想から論文投稿まで数年、また実際に掲載されるまでさらに１年から複数年かかると言われています。そういった事情もあり、博士課程の間に論文掲載に至ることはそこまで多い事例ではありません。幸いなことに、今年、悲願であった最初の１本目の論文がようやく査読付き学術誌に掲載されました。

こちらの論文は Deferred acceptance algorithm with retrade というタイトルで、東大の松井彰彦先生との共著論文です。Mathematical Social Science というゲーム理論のフィールド誌に掲載されました。研究を始めたのはおよそ６年前だったので、留学前にはモデルもおおむね完成し、定理なども書き終わっていました。しかし、本論文のモチベーションは、マッチング理論の既存文献のものとは大きく異なっていました。そのため、レフリーにモチベーションがクリアに伝わるようにイントロダクションを書くのが非常に難しく、再投稿を繰り返しながら何度も書き直すことになりました。ようやく完成をみることができ非常に感慨深いものがあります。写真は論文の１ページ目と、証明で重要となった概念図を掲載したページになります。



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Deferred acceptance algorithm with retrade☆

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ABSTRACT

We study deferred acceptance algorithm (DA) with retrade by formulating a two-stage model where DA is played in the first stage, and a decentralized market opens in the second. Both non-monetary and monetary retrades are considered. Perfect market equilibrium (PME) is defined: market equilibrium prevails in the second stage both on and off the path, and Nash equilibrium is played in the first stage game induced by the second-stage markets.

In the economies with no money, the stable and Pareto optimal allocation is the truthful PME allocation if and only if the priority structure is acyclical. A pure PME object allocation is unique if and only if the priority structure is unreversed. In the economies with money, an efficient PME allocation exists if and only if the minimum demand across tangible objects exceeds a certain threshold. A pure PME object allocation is always unique.

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1. Introduction

Mechanism with renegotiation has been studied extensively since the seminal paper by Maskin and Moore (1999). The motivation behind these studies lies in our understanding that no matter how fine a mechanism may be constructed, agents continue to play the game outside the mechanism. If the agents know the possibilities of renegotiation, then they take them into account and behave in a different manner from the situation where they do not know such possibilities. Then some desirable properties of mechanisms, such as strategy-proofness, may no longer hold.

Some analyses focus on a specific mechanism and induce more concrete results. Hafalir and Krishna (2008) consider asymmetric auctions with resale. The reason that they incorporate resale in auctions may be summarized as follows. In an asymmetric situation, a first-price auction may result in inefficient allocations. This inefficiency creates bidders' incentives for post-auction retrade. In addition, the seller may not be able to prevent bidders from engaging in the retrade even if resale was deemed disadvantageous. They show, among others, that the revenue equivalence result between first-price and second-price auctions no longer holds.

Similar post-mechanism transactions can be seen when the original mechanism is a matching algorithm instead of an auction. For example, after allocating office spaces by using a matching algorithm, we observe some post-algorithm transactions of the spaces in some universities, e.g., the Department of Economics of the University of Tokyo as well as that of Northwestern University.[1] The post-matching transactions are not governed by any planner and are carried out in a decentralized manner between the occupants of the spaces.

There are at least two reasons why retrade possibilities should be considered explicitly. The first one is positive. Suppose that post-matching transactions are inevitable. Knowing this possibility, players may have different incentives and misreport their preferences. One of the questions is how the possibility of retrade may distort the allocation achieved in the mechanism.

The second reason is normative. Suppose that the authority has sufficient resources to ban or promote decentralized transactions. Suppose further that the authority wishes to promote

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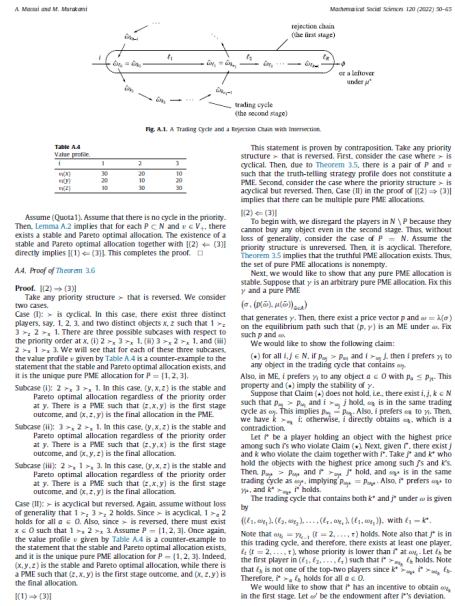
[1] In the University of Tokyo, a matching algorithm was used to allocate office spaces for faculty members, while in Northwestern University, it was used for graduate students.



A. Matsui and M. Murakami — Mathematical Social Sciences 120 (2022) 50–65

Fig. A.1. A Trading Cycle and a Rejection Chain with Intersection.

Table A.4
Value profile.

$\succ$	1	2	3
$v(\{1\})$	30	20	10
$v(\{2\})$	20	10	20
$v(\{3\})$	10	30	30

Assume (Quota1). Assume that there is no cycle in the priority. Then, Lemma A.2 implies that for each $P \subset N$ and $v \in V_-$, there exists a stable and Pareto optimal allocation. The existence of a stable and Pareto optimal allocation together with [(2) ⇐ (3)] directly implies [(1) ⇐ (3)]. This completes the proof. □

A.4. Proof of Theorem 3.6

Proof. [(2) ⇒ (3)]

Take any priority structure $\succ$ that is reversed. We consider two cases.

Case (I): $\succ$ is cyclical. In this case, there exist three distinct players, say, 1, 2, 3, and two distinct objects x, z such that $1 \succ_z 3 \succ_z 2 \succ_z 1$. There are three possible subcases with respect to the priority order at x. (i) $2 \succ_x 3 \succ_x 1$; (ii) $3 \succ_x 2 \succ_x 1$, and (iii) $2 \succ_x 1 \succ_x 3$. We will see that for each of these three subcases, the value profile v given by Table A.4 is a counter-example to the statement that the stable and Pareto optimal allocation exists, and it is the unique pure PME allocation for $P = \{1, 2, 3\}$.

Subcase (i): $2 \succ_x 3 \succ_x 1$. In this case, (y, x, z) is the stable and Pareto optimal allocation regardless of the priority order at y. There is a PME such that (z, x, y) is the first stage outcome, and (x, z, y) is the final allocation in the PME.

Subcase (ii): $3 \succ_x 2 \succ_x 1$. In this case, (y, x, z) is the stable and Pareto optimal allocation regardless of the priority order at y. There is a PME such that (z, y, x) is the first stage outcome, and (x, y, z) is the final allocation.

Subcase (iii): $2 \succ_x 1 \succ_x 3$. In this case, (y, x, z) is the stable and Pareto optimal allocation regardless of the priority order at y. There is a PME such that (z, x, y) is the first stage outcome, and (x, z, y) is the final allocation.

Case (II): $\succ$ is acyclical but reversed. Again, assume without loss of generality that $1 \succ_z 3 \succ_z 2$ holds. Since $\succ$ is acyclical, $1 \succ_x 2$ holds for all $x \in O$. Also, since $\succ$ is reversed, there must exist $x \in O$ such that $1 \succ_x 2 \succ_x 3$. Assume $P = \{1, 2, 3\}$. Once again, the value profile v given by Table A.4 is a counter-example to the statement that the stable and Pareto optimal allocation exists, and it is the unique pure PME allocation for $P = \{1, 2, 3\}$. Indeed, (x, y, z) is the stable and Pareto optimal allocation, while there is a PME such that (z, x, y) is the first stage outcome, and (x, z, y) is the final allocation.

[(1) ⇒ (3)]

This statement is proven by contraposition. Take any priority structure $\succ$ that is reversed. First, consider the case where $\succ$ is cyclical. Then, due to Theorem 3.5, there is a pair of P and v such that the truth-telling strategy profile does not constitute a PME. Second, consider the case where the priority structure $\succ$ is acyclical but reversed. Then, Case (II) in the proof of [(2) ⇒ (3)] implies that there can be multiple pure PME allocations.

[(2) ⇐ (3)]

To begin with, we disregard the players in $N \setminus P$ because they cannot buy any object even in the second stage. Thus, without loss of generality, consider the case of $P = N$. Assume the priority structure is unreversed. Then, it is acyclical. Therefore, Theorem 3.5 implies that the truthful PME allocation exists. Thus, the set of pure PME allocations is nonempty.

Next, we would like to show that any pure PME allocation is stable. Suppose that γ is an arbitrary pure PME allocation. Fix this γ and a pure PME

$$(\sigma, (p(\widehat{\omega}), \mu(\widehat{\omega}))_{\widehat{\omega} \in \Omega})$$

that generates γ. Then, there exist a price vector p and $\omega = \lambda(\sigma)$ on the equilibrium path such that (p, γ) is an ME under ω. Fix such p and ω.

We would like to show the following claim:

(⋆) for all $i, j \in N$, if $p_{\gamma_i} > p_{\omega_j}$ and $i \succ_{\omega_j} j$, then i prefers γ_i to any object in the trading cycle that contains ω_j.

Also, in ME, i prefers γ_i to any object $a \in O$ with $p_a \leq p_{\gamma_i}$. This property and (⋆) imply the stability of γ.

Suppose that Claim (⋆) does not hold, i.e., there exist $i, j, k \in N$ such that $p_{\omega_k} > p_{\omega_j}$ and $i \succ_{\omega_j} j$ hold, ω_k is in the same trading cycle as ω_j. This implies $p_{\gamma_k} > p_{\gamma_i}$. Also, i prefers ω_k to γ_i. Then, we have $k \succ_{\omega_k} i$; otherwise, i directly obtains ω_k, which is a contradiction.

Let i^* be a player holding an object with the highest price among such i's who violate Claim (⋆). Next, given i^*, there exist j and k who violate the claim together with i^*. Take i^* and k^* who hold the objects with the highest price among such j's and k's. Then, $p_{\omega_{k^*}} > p_{\gamma_{i^*}}$ and $i^* \succ_{\omega_{j^*}} j^*$ hold, and ω_{k^*} is in the same trading cycle as ω_{j^*}, implying $p_{\gamma_{k^*}} = p_{\omega_{k^*}}$. Also, i^* prefers ω_{k^*} to γ_{i^*}, and $k^* \succ_{\omega_{k^*}} i^*$ holds.

The trading cycle that contains both k^* and j^* under ω is given by

$$((\ell_1, \omega_{\ell_1}), (\ell_2, \omega_{\ell_2}), \ldots, (\ell_\tau, \omega_{\ell_\tau}), (\ell_1, \omega_{\ell_1})), \text{ with } \ell_1 = k^*.$$

Note that $\omega_{\ell_t} = \gamma_{\ell_{t-1}}$ $(t = 2, \ldots, \tau)$ holds. Note also that j^* is in this trading cycle, and therefore, there exists at least one player, ℓ_t $(t = 2, \ldots, \tau)$, whose priority is lower than i^* at ω_{ℓ_t}. Let ℓ_h be the first player in $(\ell_1, \ell_2, \ldots, \ell_\tau)$ such that $i^* \succ_{\omega_{\ell_h}} \ell_h$ holds. Note that ℓ_h is not one of the top-two players since $k^* \succ_{\omega_{k^*}} i^* \succ_{\omega_{\ell_h}} \ell_h$. Therefore, $i^* \succ_{\ell_h} \ell_h$ holds for all $a \in O$.

We would like to show that i^* has an incentive to obtain ω_{ℓ_h} in the first stage. Let ω' be the endowment after i^*'s deviation.

現在も、単著で取り組んでいた進化ゲームの研究が１本 R&R(いわゆる revise and resubmit)になっています。論文を書き直す作業は、論文をはじめに構想する作業とまた違った難しさがあると感じています。論文を構想する段階は、リサーチクエスチョンがたつかどうかと、数学的に証明できるかどうかが主に重要な点です。私の場合は理論研究なので、はじめにまず数理的なモデルを構築する必要があります。とはいえ、モデルの性質をただ調べるだけではなく、逆に、新規性のある定理を証明できるように「仕掛け」をモデルに仕込んでいくことも同時に行います。この過程で、自分が書き表したいことと、現在の分野のニーズをすり合わせながら落としどころを探ることになります。もちろん、証明したい定理が証明できる保証もないため、そこも腕の見せ所になります。これらは創造的な苦労であ

り、自分の理解が深まる知的営みなのでどちらかというと「内向き」で「自分のため」の努力です。一方で、論文を書き直す大変さは「外向き」で「他人のため」の努力に感じます。

論文投稿後、経済学では、R&R になることがほとんどです。レフリーから受け取ったレターの内容に合わせて書き直しの作業が始まります。レフリーの意図を正しく汲み、一つ一つに丁寧に答える必要があります。レフリーからモデルの拡張を求められることもあります。しかし、様々な理由でその拡張が無理な場合もあります。そういった時にはなぜできないのかを理路整然と説明する義務があります。このような地道な作業は当初想像していたよりも心理的ハードルが高く、なかなか慣れることができません。自分の中ですでに「分かっていること」を、いまだ「分かっていない」人に向けて書くというのはわかっていない人の視点に立たないとできないことだからかもしれません。そのための知恵として、一度書き上げてから数週間作業を止め、意図的に論文の内容を少し忘れてから書き直しを再開するとよい、というアドバイスを頂いたこともあります。

一人前の研究者になるためには、内容もさることながら、1 本の論文をきちんと仕上げることが何より重要だそうです。とはいえ、授業などではなかなか習うことができず、ぶっつけ本番で自分の論文を書き直しながらよりよい方法を模索していくしかありません。今はまだ戸惑いも多く、筆もなかなか進みませんが、本数を重ねる中でコツをつかんでいきたいと思います。

さて、研究とは関係ありませんが、パンデミック終結を感じさせる個人的に嬉しいサプライズがあったので話をかえてお知らせしたいと思います。アメリカでは野生のリスがそこかしこにいるのが有名です。パンデミック以前、気晴らしに私は大学のそばの公園でそういったリスを写真撮影していたことがありました。つい先日数年ぶりにその公園を訪れたところ、なんと当時なついていたリスたちがどこからともなく近寄ってきてくれました。まさかまだ私のことを覚えているとは思っていなかったのでとても感動し、心が温まる思いがしました。（私がリス一般に好かれているわけではなく、他の場所ではたとえ近寄ったとしても逃げられてしまいます。）昔のものになりますが、パンデミック前に撮影したリスの写真からいくつかこちらに紹介して今回の報告書を終えたいと思います。